

A More Precise Extraction of $|V_{cb}|$ in HQEFT of QCD

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The more precise extraction for the CKM matrix element $|V_{cb}|$ in the heavy quark effective field theory (HQEFT) of QCD is studied from both exclusive and inclusive semileptonic B decays. The values of relevant nonperturbative parameters up to order $1/m_Q^2$ are estimated consistently in HQEFT of QCD. Using the most recent experimental data for B decay rates, $|V_{cb}|$ is updated to be $|V_{cb}| = 0.0395 \pm 0.0011_{\text{exp}} \pm 0.0019_{\text{th}}$ from $B \rightarrow D^* l \nu$ decay and $|V_{cb}| = 0.0434 \pm 0.0041_{\text{exp}} \pm 0.0020_{\text{th}}$ from $B \rightarrow D l \nu$ decay as well as $|V_{cb}| = 0.0394 \pm 0.0010_{\text{exp}} \pm 0.0014_{\text{th}}$ from inclusive $B \rightarrow X_c l \nu$ decay.

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I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{cb} describes the rich phenomena of flavor-changing transitions between the two heavy quarks b and c . Its precise extraction has become a very important issue in heavy flavor physics. Generally $|V_{cb}|$ is extracted by studying either exclusive or inclusive semileptonic B decays. Since long distance contributions are involved in these decays, theoretical estimates of the nonperturbative parameters are of crucial importance for a precise extraction of $|V_{cb}|$. Heavy quark symmetry (HQS) [1–3] and effective field theories of heavy quarks play a special role in such estimates.

In this short note, we are going to provide a more precise extraction for $|V_{cb}|$ from both exclusive and inclusive semileptonic B decays within the framework of HQEFT of QCD. This effective field theory can directly be derived from QCD [4] by integrating out the small components but with carefully treating the quark and antiquark components. Recently, it has been shown that the HQEFT can actually be regarded as a large component QCD [5]. Of particular, this effective field theory has been put forward and successfully applied to various hadron processes [6–16]. The resulting effective Lagrangian and the heavy quark expansion (HQE) in the HQEFT of QCD appear to be different from the usual heavy quark effective theory (HQET) [17,18] in which the quark and antiquark components were dealt with separately, i.e., there is no quark-antiquark coupling terms considered in HQET. The application of the usual HQET in extracting $|V_{cb}|$ has been discussed by several groups and summarized in ref. [19]. We shall further emphasize in this note the relevant new features of HQEFT with respect to the usual HQET. Our paper is organized as follows: $|V_{cb}|$ extraction from exclusive and inclusive B decays will be discussed in section II and section III, respectively. The previously obtained values for $|V_{cb}|$ will be updated by using the most recent experimental measurements. And a brief conclusion will be presented in section IV.

II. $|V_{CB}|$ FROM EXCLUSIVE DECAYS

The $B \rightarrow D^*(D)l\nu$ differential decay rates are

$$\begin{aligned} \frac{d\Gamma(B \rightarrow D^* l \nu)}{d\omega} &= \frac{G_F^2}{48\pi^3} (m_B - m_{D^*})^2 m_{D^*}^3 \sqrt{\omega^2 - 1} (\omega + 1)^2 \\ &\times [1 + \frac{4\omega}{\omega + 1} \frac{m_B^2 - 2\omega m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}] |V_{cb}|^2 \mathcal{F}^2(\omega), \end{aligned} \quad (2.1)$$

$$\frac{d\Gamma(B \rightarrow D l \nu)}{d\omega} = \frac{G_F^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} |V_{cb}|^2 \mathcal{G}^2(\omega) \quad (2.2)$$

with

$$\mathcal{F}(1) = \eta_A h_{A_1}(1) = \eta_A (1 + \delta^*), \quad (2.3)$$

$$\mathcal{G}(1) = \eta_V [h_+(1) - \frac{m_B - m_D}{m_B + m_D} h_-(1)] = \eta_V (1 + \delta), \quad (2.4)$$

where the QCD radiative corrections to two loops give the short distance coefficients $\eta_A = 0.960 \pm 0.007$ and $\eta_V = 1.022 \pm 0.004$ [20]. ω is the product of the four-velocities of the B and $D^*(D)$ mesons, $\omega = v \cdot v'$. The weak transition form factors $h_{A_1}(\omega)$, $h_+(\omega)$ and $h_-(\omega)$ can be expanded in powers of $1/m_Q$ and represented by the heavy quark spin-flavor independent wave functions. Then based on Eqs.(2.1) and (2.2), $|V_{cb}|$ can be precisely extracted as long as the form factors are reliably evaluated in some theoretical framework.

The HQE of the transition form factors are studied in detail up to order $1/m_Q^2$ in HQEFT framework in Ref. [6]. When the contributions of operators containing two gluon field strength tensors are omitted, at the zero recoil point $\omega = 1$ the relevant form factors can be written as

$$\begin{aligned} h_{A_1} = & 1 + \frac{1}{8\bar{\Lambda}^2} \left[\frac{1}{m_b}(\kappa_1 + 3\kappa_2) - \frac{1}{m_c}(\kappa_1 - \kappa_2) \right]^2 - \frac{1}{8m_b^2\bar{\Lambda}^2}(F_1 + 3F_2 \\ & - 2\bar{\Lambda}\varrho_1 - 6\bar{\Lambda}\varrho_2) - \frac{1}{8m_c^2\bar{\Lambda}^2}(F_1 - F_2 - 2\bar{\Lambda}\varrho_1 + 2\bar{\Lambda}\varrho_2) \\ & + \frac{1}{4m_b m_c \bar{\Lambda}^2}(F_1 + F_2 - 2\bar{\Lambda}\varrho_1 - 2\bar{\Lambda}\varrho_2), \end{aligned} \quad (2.5)$$

$$h_+ = 1 + \frac{1}{8\bar{\Lambda}^2} \left(\frac{1}{m_b} - \frac{1}{m_c} \right)^2 [(\kappa_1 + 3\kappa_2)^2 - (F_1 + 3F_2) + 2\bar{\Lambda}(\varrho_1 + 3\varrho_2)], \quad (2.6)$$

$$h_- = 0, \quad (2.7)$$

where the parameters in the rhs. of Eqs.(2.5)-(2.7) are defined in HQEFT [6]. For simplicity, when the variable ω is not written explicitly, we refer to the zero recoil values of relevant functions, i.e., $h_{A_1} = h_{A_1}(1)$, $\kappa_1 = \kappa_1(1)$, etc. The binding energy of a heavy meson M is defined as

$$\bar{\Lambda} \equiv \lim_{m_Q \rightarrow \infty} \bar{\Lambda}_M = \lim_{m_Q \rightarrow \infty} (m_M - m_Q). \quad (2.8)$$

It is seen from Eqs.(2.5) and (2.6) that automatically both the form factors h_{A_1} and h_+ in HQEFT of QCD do not receive $1/m_Q$ order correction at the zero recoil point. Another point to be emphasized is Eq.(2.7). The vanished value of h_- in HQEFT arises from the fact of partial cancellation between the $1/m_Q$ correction in the current expansion and the $1/m_Q$ correction coming from the insertion of the effective Lagrangian into the transition matrix elements [6]. Such a cancellation is not observed in HQET because in the latter framework the quark-antiquark couplings are not taken into account explicitly. Though the so-called Luke's theorem in HQET protects the weak transition matrix elements from $1/m_Q$ order correction at zero recoil point, it does not protect h_- from such correction [21]. As a result, in HQET framework, only the $B \rightarrow D^* l \nu$ decay rate at zero recoil is strictly protected against $1/m_Q$ order correction while $B \rightarrow D l \nu$ decay rate is not. This is the main reason besides the experimental considerations for the conclusion that the $B \rightarrow D l \nu$ decay is not as favorable as $B \rightarrow D^* l \nu$ decay for $|V_{cb}|$ extraction in HQET.

In HQEFT, Eq.(2.7) advocates reliable extraction of $|V_{cb}|$ from both $B \rightarrow D^* l \nu$ and $B \rightarrow D l \nu$ decays, because both rates of these decays do not receive $1/m_Q$ order correction, as can be seen from Eqs.(2.1)-(2.7).

However, we note that the current world averages for $|V_{cb}|\mathcal{F}(1)$ and $|V_{cb}|\mathcal{G}(1)$ are [19]

$$|V_{cb}|\mathcal{F}(1) = 0.0383 \pm 0.0005 \pm 0.0009, \quad (2.9)$$

$$|V_{cb}|\mathcal{G}(1) = 0.0413 \pm 0.0029 \pm 0.0027, \quad (2.10)$$

so the current experimental data for $|V_{cb}|\mathcal{G}(1)$ receive larger errors than those for $|V_{cb}|\mathcal{F}(1)$, which may lead to large experimental uncertainty for the result of $|V_{cb}|$ extracted from $B \rightarrow D l \nu$.

Now the value of $|V_{cb}|$ depends on our estimates of $\mathcal{F}(1)$ and $\mathcal{G}(1)$. First of all, it has been shown [6] that some of the nonperturbative parameters appearing in the rhs. of Eqs.(2.5) -(2.7) can be related to the heavy meson masses. Explicitly, we have

$$\bar{\Lambda}_{D(B)} = \bar{\Lambda} - \left(\frac{1}{m_{c(b)}} - \frac{\bar{\Lambda}}{2m_{c(b)}^2} \right) (\kappa_1 + 3\kappa_2) - \frac{1}{4m_{c(b)}^2\bar{\Lambda}} (F_1 + 3F_2) + O\left(\frac{1}{m_{c(b)}^3}\right), \quad (2.11)$$

$$\bar{\Lambda}_{D^*(B^*)} = \bar{\Lambda} - \left(\frac{1}{m_{c(b)}} - \frac{\bar{\Lambda}}{2m_{c(b)}^2} \right) (\kappa_1 - \kappa_2) - \frac{1}{4m_{c(b)}^2\bar{\Lambda}} (F_1 - F_2) + O\left(\frac{1}{m_{c(b)}^3}\right). \quad (2.12)$$

Thus some parameters can be determined from the heavy meson masses. A detailed study has been presented in Ref. [6]. Here we would not repeat the analysis. It is easily read from Ref. [6] that by using

$$m_b = 4.7\text{GeV}, \quad m_b - m_c = 3.36\text{GeV} \quad m_b + \bar{\Lambda} = 5.21\text{GeV}, \quad (2.13)$$

one obtains the following values for κ_1 , κ_2 and F_1 , F_2 :

$$\begin{aligned} \kappa_1 &\approx -0.615\text{GeV}^2, \quad \kappa_2 \approx 0.056\text{GeV}^2, \\ F_1 &\approx 0.917\text{GeV}^4, \quad F_2 \approx 0.004\text{GeV}^4, \end{aligned} \quad (2.14)$$

which agree well with the sum rule results: $\kappa_1 = -0.5 \pm 0.18\text{GeV}^2$, $\kappa_2 \approx 0.08\text{GeV}^2$ and $\bar{\Lambda} = 0.53 \pm 0.08\text{GeV}$ [8].

Then up to order $1/m_Q^2$ only two parameters ϱ_1 and ϱ_2 remain unknown. To have an estimation for them, we recall the definition of the nonperturbative parameters. They are defined by the matrix elements as follows [6]:

$$\begin{aligned} \langle M'_{v'} | \bar{Q}_{v'}^{(+)} \Gamma \frac{1}{iv \cdot D} (i\not{D}_\perp)^2 Q_v^{(+)} | M_v \rangle &= -\kappa_1(\omega) \frac{1}{\bar{\Lambda}} \text{Tr}[\bar{\mathcal{M}}' \Gamma \mathcal{M}] \\ &\quad + \frac{1}{\bar{\Lambda}} \text{Tr}[\kappa_{\alpha\beta}(v, v') \bar{\mathcal{M}}' \Gamma P_+ \frac{i}{2} \sigma^{\alpha\beta} \mathcal{M}], \\ \langle M'_{v'} | \bar{Q}_{v'}^{(+)} \Gamma \frac{1}{iv \cdot D} (i\not{D}_\perp) (-iv \cdot D) (i\not{D}_\perp) Q_v^{(+)} | M_v \rangle &= -\varrho_1(\omega) \frac{1}{\bar{\Lambda}} \text{Tr}[\bar{\mathcal{M}}' \Gamma \mathcal{M}] \\ &\quad + \frac{1}{\bar{\Lambda}} \text{Tr}[\varrho_{\alpha\beta}(v, v') \bar{\mathcal{M}}' \Gamma P_+ \frac{i}{2} \sigma^{\alpha\beta} \mathcal{M}], \\ \langle M'_{v'} | \bar{Q}_{v'}^{(+)} \Gamma \frac{1}{iv \cdot D} (i\not{D}_\perp)^2 \frac{1}{iv \cdot D} (i\not{D}_\perp)^2 Q_v^{(+)} | M_v \rangle &= -\chi_1(\omega) \frac{1}{\bar{\Lambda}^2} \text{Tr}[\bar{\mathcal{M}}' \Gamma \mathcal{M}] \\ &\quad + \frac{1}{\bar{\Lambda}^2} \text{Tr}[\chi_{\alpha\beta}(v, v') \bar{\mathcal{M}}' \Gamma P_+ \frac{i}{2} \sigma^{\alpha\beta} \mathcal{M}] + \dots, \end{aligned} \quad (2.15)$$

where the ellipsis in the last equation represents the contributions of operators containing two gluon field strength tensors. The Lorentz tensor $\kappa_{\alpha\beta}$ is decomposed into scalar factors as $\kappa_{\alpha\beta}(v, v') = i\kappa_2(\omega)\sigma_{\alpha\beta} + \kappa_3(\omega)(v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha)$, and $\varrho_{\alpha\beta}(v, v')$, $\chi_{\alpha\beta}(v, v')$ are decomposed similarly. The functions F_1 and F_2 are defined as

$$F_1 = \chi_1 + 2\bar{\Lambda}\varrho_1, \quad (2.16)$$

$$F_2 = \chi_2 + 2\bar{\Lambda}\varrho_2. \quad (2.17)$$

As will be further emphasized in the next section, the operator $iv \cdot D$ in the matrix elements was considered to give contribution of order the binding energy [6,7], i.e., $iv \cdot D \sim v \cdot k \sim \bar{\Lambda}^{-1}$, which has also been confirmed in the simple case by sum rule calculation [8]. Now if taking this simple replacement, we get from Eqs.(2.15)

$$\varrho_1 \approx -\kappa_1 \cdot \bar{\Lambda} \approx 0.3\text{GeV}^3, \quad \chi_1 \approx \kappa_1^2 \approx 0.4\text{GeV}^4. \quad (2.18)$$

Note that our estimates in Eqs.(2.14) and (2.18) are consistent with the relation among them in Eq.(2.16).

In Eq.(2.14) F_2 almost equals zero. Of course one reason for this may be the possible partial cancellation between χ_2 and $2\bar{\Lambda}\varrho_2$ in Eq.(2.17). But more importantly, we notice that χ_2 , ϱ_2 (and therefore F_2) are parameters characterizing the chromomagnetic type operators, and the contributions of these chromomagnetic operators are generally believed to be small.

If using the same method of estimation as that for ϱ_1 and χ_1 , one gets

$$\varrho_2 \sim -\kappa_2 \cdot \bar{\Lambda} \approx -0.03\text{GeV}^3, \quad (2.19)$$

$$\chi_2 \sim \kappa_2^2 \approx 0.003\text{GeV}^4. \quad (2.20)$$

It can be seen from the above equations that the two terms in Eq.(2.17) do have partial cancellation. However, Eqs.(2.19), (2.20) and (2.17) can not hold simultaneously. This indicates that Eqs.(2.19) and (2.20) are only rough estimates for the chromomagnetic type parameters since these parameters are very small in magnitude. Nevertheless, as ϱ_2 must be small and can not influence the final result for $|V_{cb}|$

¹This relation is in the sense of the effective contributions of operators within some matrix element.

as significantly as ϱ_1 does, here we would first take Eq.(2.19) in extracting $|V_{cb}|$ but leave more rigid determination of ϱ_2 for future work.

Fig.1 presents δ^* as a function of $m_b + \bar{\Lambda}$ at the fixed quark mass difference $m_b - m_c = 3.36\text{GeV}$. The curves are relatively flat in the region around $m_b + \bar{\Lambda} = 5.2\text{GeV}$, which indicates a reliable extraction of δ^* around this point. This value of $m_b + \bar{\Lambda}$ is in agreement with that in Eq.(2.14). Varying m_b and $m_b - m_c$ in the ranges $4.6\text{GeV} \leq m_b \leq 4.8\text{GeV}$ and $3.32\text{GeV} \leq m_b - m_c \leq 3.41\text{GeV}$, and taking $\varrho_1 \approx 0.3\text{GeV}^3$ and $\varrho_2 \approx -0.03\text{GeV}^3$, we get

$$\delta^* = 0.01 \pm 0.04. \quad (2.21)$$

From Eqs.(2.9) and (2.21), we then obtain

$$|V_{cb}| = 0.0395 \pm 0.0011_{\text{exp}} \pm 0.0019_{\text{th}}. \quad (2.22)$$

For $B \rightarrow D l \nu$ decay, analogously, we obtain

$$\delta = -0.07 \pm 0.04 \quad (2.23)$$

and

$$|V_{cb}| = 0.0434 \pm 0.0041_{\text{exp}} \pm 0.0020_{\text{th}}. \quad (2.24)$$

Comparing Eqs.(2.22) and (2.24), we see that the results of $|V_{cb}|$ extracted from $B \rightarrow D^*(D) l \nu$ decays have similar theoretical errors. But large difference exists between the central values of our present results, Eqs.(2.22) and (2.24). However, due to the large experimental error for $B \rightarrow D l \nu$ channel, the $|V_{cb}|$ values extracted above are compatible with each other. A better determination of $|V_{cb}|$ from $B \rightarrow D l \nu$ decay is expected when a more precise result for the quantity $|V_{cb}| \mathcal{G}(1)$ is obtained.

Note that in Ref. [6] ϱ_1 and ϱ_2 are not estimated as above. There $\varrho_2 \approx 0.1\text{GeV}^3$ is assumed so that the central values for $|V_{cb}|$ extracted from $B \rightarrow D^*(D) l \nu$ decays can be close to each other. Indeed, $\varrho_2 \approx 0.1\text{GeV}^3$ together with the experimental averages (2.9), (2.10) lead to $|V_{cb}| = 0.0416 \pm 0.0011_{\text{exp}} \pm 0.0016_{\text{th}}$ from $B \rightarrow D^* l \nu$ decay and $|V_{cb}| = 0.0417 \pm 0.0040_{\text{exp}} \pm 0.0019_{\text{th}}$ from $B \rightarrow D l \nu$ decay.

III. $|V_{CB}|$ FROM INCLUSIVE DECAYS

Inclusive semileptonic B decays is the other alternative to determine $|V_{cb}|$. In the usual HQET the light quark in a hadron is generally treated as a spectator, which does not affect the heavy hadron properties to a large extent. However, this treatment may be one possible reason for the failure of HQET in some applications. For example, the world average value for bottom hadron lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ can not be explained well in the usual framework of HQET [22,23].

The dynamics of inclusive B decays is also analyzed in detail in Refs. [7,9] within the HQEFT framework. Instead of simply applying the equation of motion for infinitely heavy free quark, $i v \cdot D Q_v^{(+)} = 0$, we treat the heavy quark in a hadron as a dressed particle, which means that the residual momentum k of the heavy quark within a hadron is considered to comprise contributions from the light degrees of freedom. This simple picture is adopted to conveniently take into account the effects of light degrees of freedom and the binding effects of heavy and light components of the hadron but not deal with the complex dynamics of hadronization directly. As the light degrees of freedom within the heavy hadron is relativistic, one has

$$k^0 \sim |\mathbf{k}|. \quad (3.1)$$

Explicitly, the momentum of a heavy hadron H is represented as $P_H = m_Q v + k + k'$ with k' being the momentum depending on the heavy flavor and suppressed by the inverse of the heavy quark mass. This directly leads to the relation between $v \cdot k$ and the binding energy,

$$\bar{\Lambda} = \lim_{m_Q \rightarrow \infty} \bar{\Lambda}_H = \lim_{m_Q \rightarrow \infty} (m_H - m_Q) = v \cdot k, \quad (3.2)$$

or

$$\langle i v \cdot D \rangle \equiv \frac{\langle H_v | \bar{Q}_v^{(+)} i v \cdot D Q_v^{(+)} | H_v \rangle}{2 \bar{\Lambda}} \approx \bar{\Lambda} \neq 0. \quad (3.3)$$

To acquire a good convergence of HQE, we perform the expansion in terms of $k - v(iv \cdot D)$ (or say, equivalently, in terms of $1/(m_Q + \bar{\Lambda})$). Then the $B \rightarrow X_c l \nu$ decay rate is found to be [7,9]

$$\Gamma(B \rightarrow X_c l \nu) = \frac{G_F^2 \hat{m}_b^5 V_{cb}^2}{192\pi^3} \eta_{cl}(\rho, \rho_l, \mu) \{I_0(\rho, \rho_l, \hat{\rho}) + I_1(\rho, \rho_l, \hat{\rho}) \frac{\kappa_1}{3\hat{m}_b^2} - I_2(\rho, \rho_l, \hat{\rho}) \frac{\kappa_2}{\hat{m}_b^2}\}, \quad (3.4)$$

where $\hat{m}_b = m_b + \bar{\Lambda}$, I_0 , I_1 and I_2 are functions of the mass square ratios $\rho = m_c^2/\hat{m}_b^2$, $\hat{\rho}^2 = \hat{m}_c^2/\hat{m}_b^2$ and $\rho_l^2 = m_l^2/\hat{m}_b^2$. Here the calculation is performed up to nonperturbative order $1/\hat{m}_Q^2$ and perturbative order α_s^2 . The function η_{cl} characterizes QCD radiative corrections. Its two-loop results were obtained in Refs. [24,25]. \hat{m}_b and \hat{m}_c can be determined from the meson masses via Eq.(2.11). κ_2 is often extracted from the known $B - B^*$ mass splitting

$$\kappa_2 \simeq \frac{1}{8}(m_{B^{*0}}^2 - m_{B^0}^2) \approx 0.06 \text{GeV}^2, \quad (3.5)$$

which is consistent with Eq.(2.14) and the sum rule result [8].

There are several points to be mentioned for Eq.(3.4). Firstly, in deriving Eq.(3.4) the effects of light degrees of freedom are explicitly accounted for in the picture of a dressed heavy quark in a hadron. Secondly, it is seen that the next leading order contributions vanish in our HQE in terms of the inverse dressed heavy quark mass, $1/\hat{m}_b$. Furthermore, our HQE in terms of $k - v(iv \cdot D)$ (or $1/\hat{m}_b$) has a good convergence. It is found that the $1/\hat{m}_b^2$ order contributions induce only $-0.7 \sim 5\%$ corrections to the total width $\Gamma(H_b \rightarrow X_c e \bar{\nu})$. Therefore we conclude that the higher order nonperturbative corrections can be safely neglected. Finally, now one needs only to treat the dressed quark mass $\hat{m}_b = m_b + \bar{\Lambda}$ instead of considering the uncertainties arising from the two quantities m_b and $\bar{\Lambda}$ separately. Note that these are the features of HQE in HQEFT. They can not be observed in the HQE in the usual HQET, where one assumes $\langle iv \cdot D \rangle$ to be zero or of higher order of $1/m_b$. In the latter case the next leading order corrections can be absent only when the HQE is performed in terms of $1/m_b$, and the heavy quark mass m_b and the binding energy $\bar{\Lambda}$ have to be treated separately. One result of this is that in HQET the theoretical prediction of the total decay width strongly depends on the value of bottom quark mass m_b and may have larger uncertainties than in HQEFT.

Using Eq.(3.4), $|V_{cb}|$ can be extracted from experimental data for inclusive decay rates. Fig.3 shows the obtained values of $\delta_{in} \equiv \Gamma(B \rightarrow X_c l \nu)/(|V_{cb}|^2) \times 10^{11}$ as a function of the energy scale μ and the parameters m_c , κ_1 . It is seen that the extracted value of $|V_{cb}|$ depends on the energy scale μ weakly. So the main uncertainties come from m_c and κ_1 . The curves in Fig.3(b) and Fig.3(c) have minimal value points, around which the values of δ_{in} are favorable because they are less sensitive to the variation of m_c and κ_1 . Varying m_c and κ_1 in the regions $1.45 \text{GeV} < m_c < 1.85 \text{GeV}$, $-0.8 \text{GeV}^2 < \kappa_1 < -0.4 \text{GeV}^2$, these favorable values of δ_{in} change in the range:

$$\delta_{in} = 2.88 \pm 0.20 \text{ps}^{-1}, \quad (3.6)$$

where the central value is obtained at $m_c = 1.65 \text{GeV}$ and $\kappa_1 = -0.6 \text{GeV}^2$.

Then the B^0 lifetime $\tau(B^0) = 1.540 \pm 0.014 \text{ps}$ and the most recent CLEO data for $B \rightarrow X_c e \nu$ branching ratio $\text{Br}(B \rightarrow X_c e \nu) = (10.49 \pm 0.17 \pm 0.43)\%$ [26] yield

$$|V_{cb}| = 0.0394 \pm 0.0010_{\text{exp}} \pm 0.0014_{\text{th}}. \quad (3.7)$$

$|V_{cb}|$ extracted using different values of branching ratios is presented in Fig.4. As mentioned above, the values at $m_c \approx 1.65 \text{GeV}$ should be favorable because in this region the curves become less sensitive to the mass m_c . Interestingly, as shown in Fig.3(c), when choosing $m_c = 1.65 \text{GeV}$ we find that the curve of δ_{in} (or the resultant $|V_{cb}|$) as a function of $-\kappa_1$ reaches its minimal (or maximal) point at $\kappa_1 \approx -0.6 \text{GeV}^2$. This value for κ_1 is again in good agreement with that we used in section II and with that obtained from sum rule calculation [8].

Note that the mass m_c discussed here arises from the charm quark propagator in the HQE of the matrix elements. It should be the pole mass of charm quark. Thus it is not surprising that the value of m_c obtained here is larger than the value for constituent mass quoted in section II.

IV. CONCLUSION

We have presented a more precise extraction of $|V_{cb}|$ by studying the exclusive and inclusive semileptonic B decays up to the order of $1/m_Q^2$ in HQEFT of QCD. The nonperturbative parameters in HQEFT up to the same order are estimated consistently from various considerations.

It has been shown that in HQEFT of QCD $|V_{cb}|$ can be reliably extracted from exclusive decays $B \rightarrow D^*(D)l\nu$ with similar theoretical uncertainties, because neither of their differential decay rates receives $1/m_Q$ order corrections at zero recoil point. In studying inclusive B decays, we treat the heavy quark in a hadron as a dressed particle whose residual momentum comprises some effects from the light degrees of freedom. This enables us to consider the effects of light components in the hadron but still has a simple physical picture in application. In this framework, the $B \rightarrow X_cl\nu$ total decay rate is protected from $1/\hat{m}_b$ order correction, and the $1/\hat{m}_b^2$ order correction is very small. Furthermore, m_b and $\bar{\Lambda}$ only appear in the form of dressed quark mass: $\hat{m}_b = m_b + \bar{\Lambda}$, which must also reduce the uncertainties in our calculations as $\hat{m}_b = m_H[1 + O(1/m_Q^2)]$.

Using the most recent experimental data for the B-meson decay rates, we have arrived at the determination for $|V_{cb}|$ with

$$\begin{aligned} |V_{cb}| &= 0.0395 \pm 0.0011_{\text{exp}} \pm 0.0019_{\text{th}} && \text{from } B \rightarrow D^*l\nu, \\ |V_{cb}| &= 0.0434 \pm 0.0041_{\text{exp}} \pm 0.0020_{\text{th}} && \text{from } B \rightarrow Dl\nu, \\ |V_{cb}| &= 0.0394 \pm 0.0010_{\text{exp}} \pm 0.0014_{\text{th}} && \text{from } B \rightarrow X_cl\nu, \end{aligned}$$

where the result extracted from $B \rightarrow Dl\nu$ decay receives a larger experimental uncertainty than that from $B \rightarrow D^*l\nu$ decay but a similar theoretical uncertainty as the latter. The result obtained from $B \rightarrow D^*l\nu$ decay agrees quite well with that from inclusive $B \rightarrow X_cl\nu$ decay.

These results then give the average

$$|V_{cb}| = 0.0402 \pm 0.0014_{\text{exp}} \pm 0.0017_{\text{th}}. \quad (4.1)$$

Alternatively it can be represented as

$$A = 0.83 \pm 0.07 \quad (4.2)$$

in the Wolfenstein parameterization $|V_{cb}| = A\lambda^2$ with $\lambda = |V_{us}| = 0.22$.

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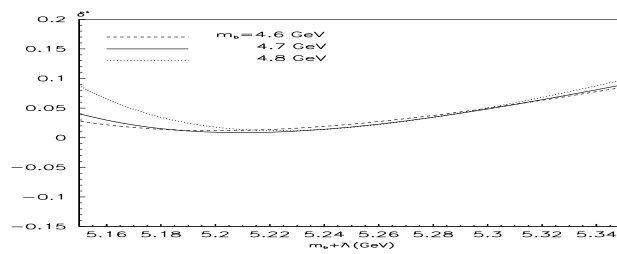


FIG. 1. δ^* as a function of $m_b + \bar{\Lambda}$ at $m_b - m_c = 3.36\text{GeV}$.

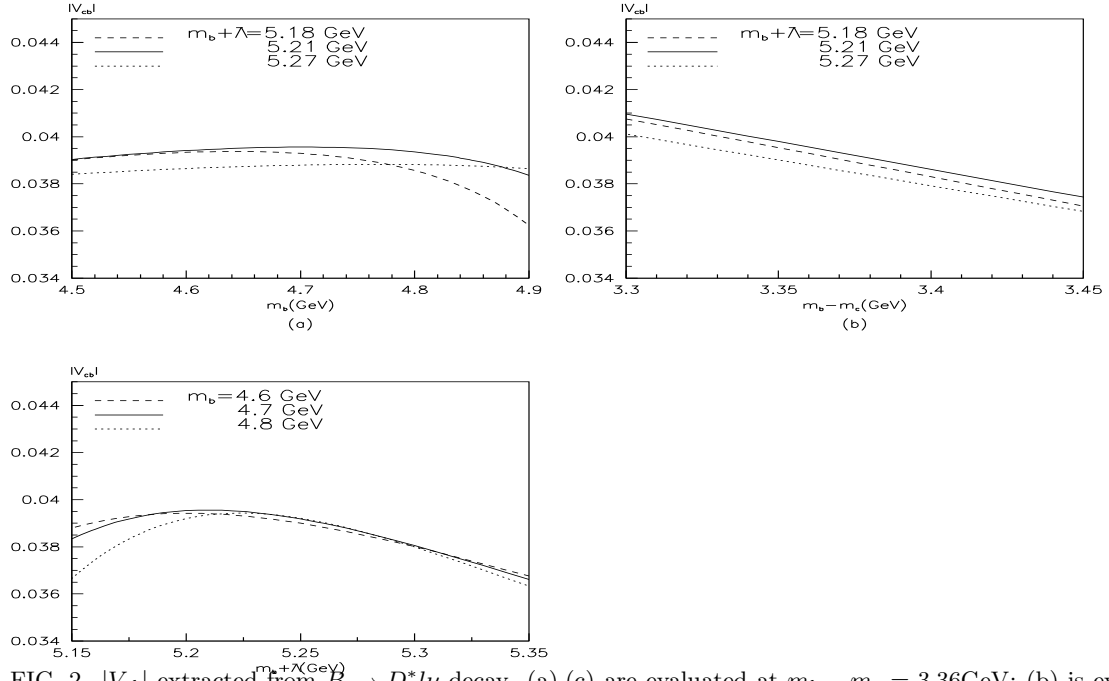


FIG. 2. $|V_{cb}|$ extracted from $B \rightarrow D^* l \nu$ decay. (a),(c) are evaluated at $m_b - m_c = 3.36$ GeV; (b) is evaluated at $m_b = 4.7$ GeV.

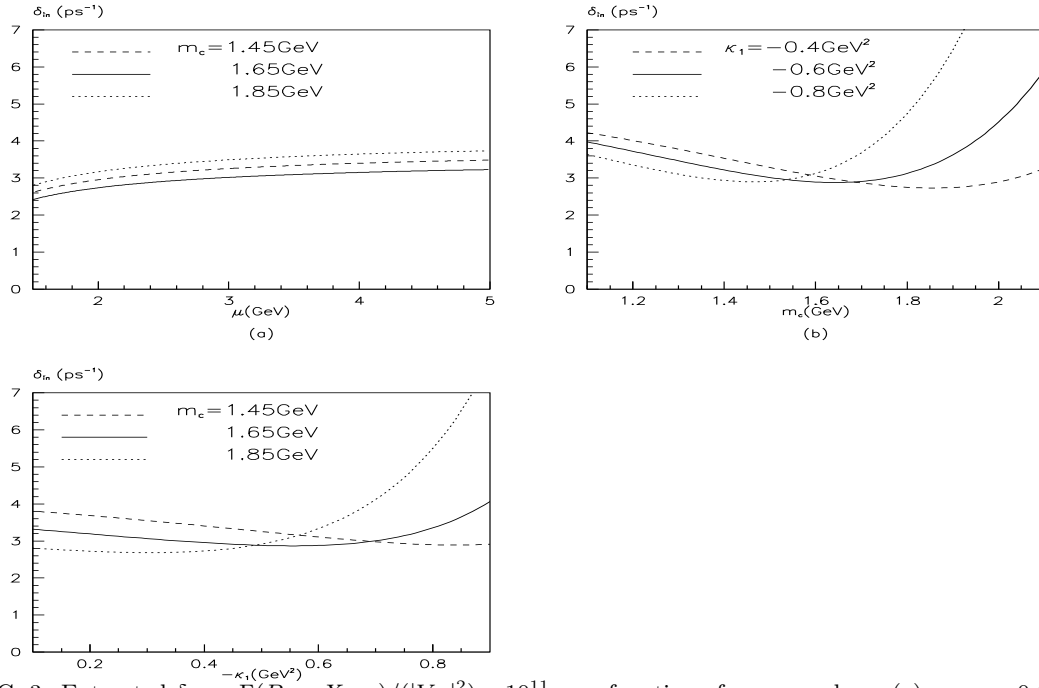


FIG. 3. Extracted $\delta_{in} \equiv \Gamma(B \rightarrow X_c e \nu) / (|V_{cb}|^2) \times 10^{11}$ as a function of μ , m_c and κ_1 . (a): $\kappa_1 = -0.6 \text{ GeV}^2$; (b) and (c): $\mu = 2.4 \text{ GeV}$.

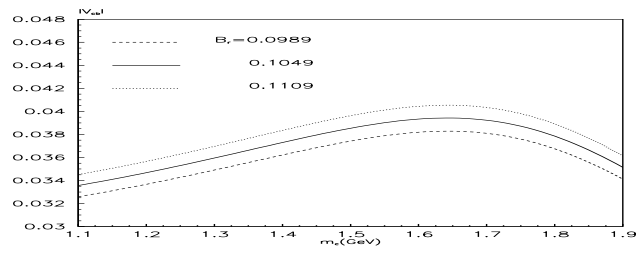


FIG. 4. $|V_{cb}|$ extracted from $B \rightarrow X_c e \nu$ decay at $\mu = 2.4 \text{ GeV}$ and $\kappa_1 = -0.6 \text{ GeV}^2$.